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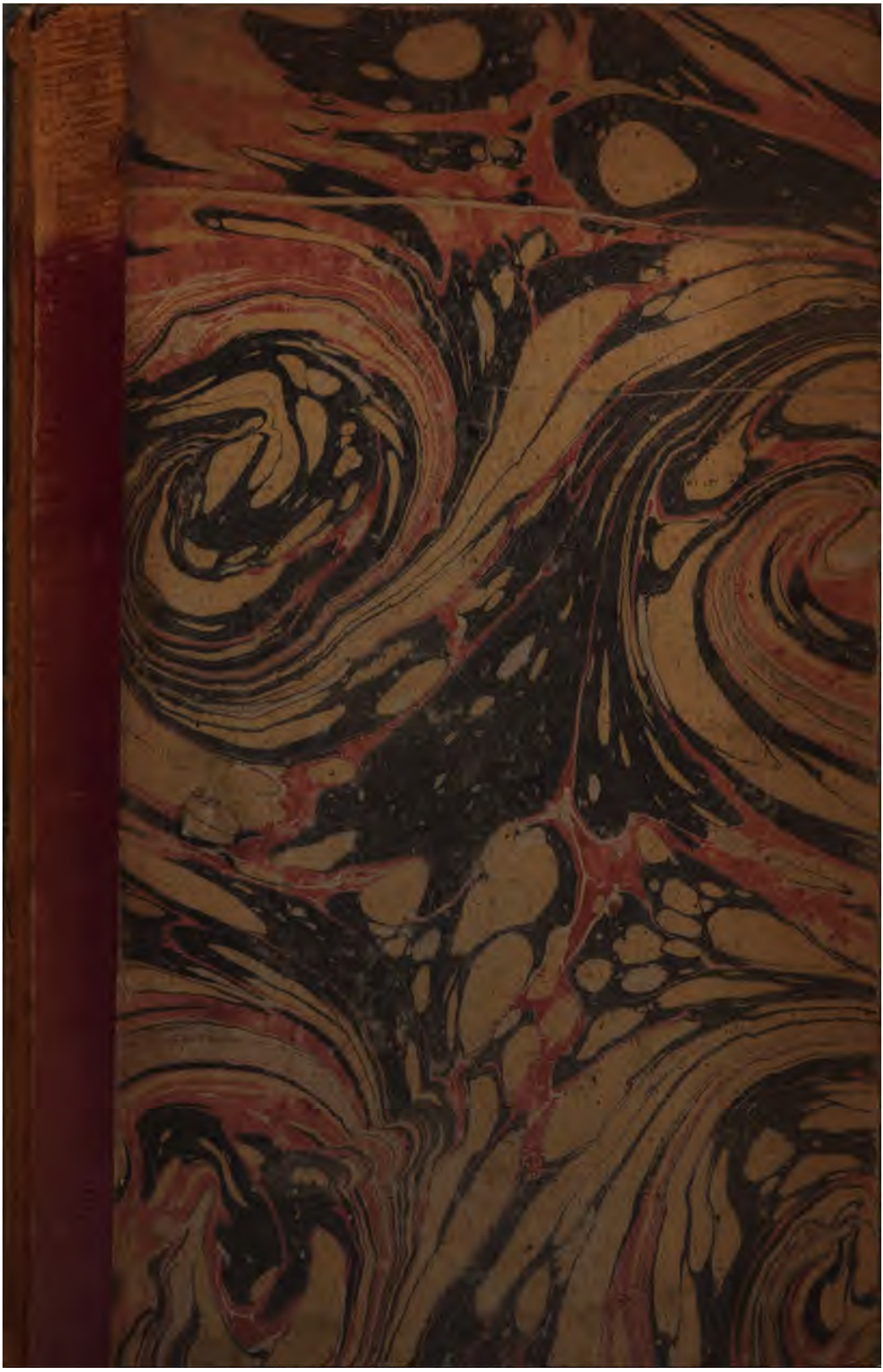

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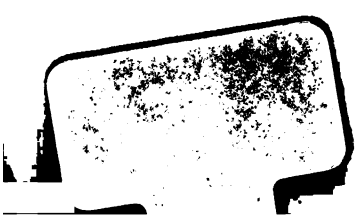

CAMBRIDGE PROBLEMS,

PROPOSED BY THE MODERATORS

TO THE

CANDIDATES FOR MATHEMATICAL HONORS,

AT THE

GENERAL EXAMINATION, 1843.

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CAMBRIDGE PROBLEMS.

WEDNESDAY, *January* 4, 1843.

QUESTIONS IN PURE MATHEMATICS.

N.B.—The Differential Calculus is not to be used.

1. THE opposite sides and angles of parallelograms are equal to one another, and the diameter bisects them, that is, divides them into two equal parts.

2. In a circle the angle in a semi-circle is a right angle; but the angle in a segment greater than a semi-circle is less than a right angle; and the angle in a segment less than a semi-circle is greater than a right angle.

3. What sum must be invested in the 3 per cent. stock at $94\frac{1}{4}$, to yield an annual income of £500?

4. Shew how to find the least common multiple of two quantities: and extend the method to find that of several quantities.

Ex. Find the least common multiple of $x^3 - x$, $x^3 - 1$, $x^3 + 1$.

5. Find the sum of any number of terms of a series in arithmetic progression.

The sum of an arithmetic progression is 72, the first term is 24, and the common difference -4, find the number of terms.

6. Solve the equations:

(1) $\frac{2x-6}{5} - \frac{x-4}{9} - \frac{3x}{13} = 0.$ (2) $x^4 - 14x^2 + 1 = 0.$

(3) $$\begin{aligned} a\,x + b\,y + c\,z &= d, \\ a_1 x + b_1 y + c_1 z &= d_1, \\ a_2 x + b_2 y + c_2 z &= d_2. \end{aligned}$$

Find when equations (3) are equivalent to only two independent equations.

7. The coefficients of terms equidistant from the ends, in the expansion of $(1 + x)^n$ (n a positive integer) are equal.

Write down the general term of $\frac{1}{\sqrt{1+x}}$.

8. Shew that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

Prove that $\tan(45^0 + A) - \tan(45^0 - A) = 2 \tan 2A$. What is the use of the latter formula?

9. Explain the ambiguous case of plane triangles, giving a geometrical illustration of it. When $C = 30^0$, $b = 16$, $c = 8$, is the triangle ambiguous or not?

10. Find the angle between two straight lines of which the equations are given.

Find the equations to lines passing through a given point, and inclined at an angle of 45^0 to the line $y = x + a$.

11. Find the equation to a straight line touching a parabola at a given point.

Find the points of contact of tangents, the perpendiculars on which from the foot of the directrix, shall be equal to one-fourth of the latus rectum.

12. Define conjugate diameters of an ellipse. Shew that there can be but one pair of rectangular conjugate diameters.

13. Shew how to reduce the reciprocal equation

$$x^{2n} - px^{2n-1} + \ldots = 0$$

which has no roots equal to unity.

If the terms be alternately positive and negative, and p not greater than $2n$, the roots cannot all be real.

14. In isosceles spherical triangles, the angles at the base are equal.

15. Prove the formula for finding the cosine of an angle of a spherical triangle in terms of the sides.

16. Shew how to construct a table of natural sines.

Given cosine $34^0 . 29' = 4338335$, cosine $34^0 . 30' = 4335938$.

Find the cosine of $34^0 . 29' . 48''$.

WEDNESDAY, *January* 4.

QUESTIONS IN NATURAL PHILOSOPHY.

N.B.—The Differential Calculus is not to be used.

1. ASSUMING the parallelogram of forces, determine the resultant of any number of forces in the same plane acting on a point.

At any point of the circumference of a circle two equal forces act in directions passing through two fixed points on the circumference, shew that the resultant of these forces always passes through a fixed point.

2. Find the ratio of the power to the weight when there is equilibrium on the wheel and axle.

What mechanical powers are reducible to the principle of the lever?

3. Define the centre of gravity of a body. Find the centre of gravity of a quadrilateral figure two sides of which are parallel.

4. How is force measured in Dynamics? What is denoted by the letter (g) in dynamical formulæ? If the unit of time be one minute, what is the measure of the accelerating force of gravity? How are the Statical and Dynamical measures of force connected?

5. State the principles on which is founded the theory of the direct impact of elastic bodies. Two equal and imperfectly elastic bodies impinge directly: having given the original velocity of the one, find that of the other, in order that after the blow it may be reduced to rest.

6. Determine the directions in which a particle must be projected with a given velocity, in order that it may hit a mark at a given distance on the same horizontal line with the point of projection.

7. A body floats in water, find the conditions of equilibrium.

A cylinder with its axis vertical floats in two fluids of different densities, find the ratio of the two parts into which the cylinder is divided by the common surface of the two fluids.

8. Describe the common low pressure steam-engine. Is it necessary that the steam should be admitted into the cylinder during the whole stroke?

9. By what experiments is it proved that the pressure of air is proportional to its density, the temperature being constant? If the temperature vary, what is the law of variation of the pressure?

10. Shew that the image of an object in a plane mirror is equal to the object, and equally inclined to the mirror.

11. Find the limit of the refracting angle of a prism of given refractive power so that the passage of a ray of light through it may be possible.

12. Describe the construction of the simple astronomical telescope, shewing on what depend its magnifying power, and the brightness of the image.

If the instrument be adjusted for an ordinary eye, what change must be made to suit a short sighted one?

13. State and prove Newton's seventh Lemma.

By what artifice has Newton avoided the use of indefinitely small quantities in this and the two following Lemmas?

14. If any number of bodies revolve in ellipses round the same centre of force varying inversely as the square of the distance, the squares of the periodic times are proportional to the cubes of the major axes of the orbits.

15. Define the terms "meridian," "terrestrial latitude and longitude," and "ecliptic."

Shew that in latitude 66°. 32′ the longest is 24 hours.

16. What are the three usual modes of measuring time? What time does the astronomical clock shew? What time does a dial shew? Find the mean time of a given star's transit.

17. Describe the vernier. If the limb of the instrument is divided to 10′, and the vernier reads to 10″, how many divisions are there in the vernier?

THURSDAY, *January* 5.

QUESTIONS IN NATURAL PHILOSOPHY.

1. Two parallel forces act in opposite directions on a rigid body, find the magnitude of the resultant and the line in which it acts.

When the forces become equal interpret the result.

2. Adapt the general formulæ for finding the centre of gravity of a body to the case of a solid revolution.

A solid sector of a sphere hangs from a point in its circular rim with its axis horizontal, find its vertical angle.

3. A body moves round a centre of force varying inversely as the square of the distance, shew that the species of conic section described is independent of the direction of projection.

4. State Kepler's laws. Shew from the first that the force on the planets tends towards the sun. State the theoretical deductions from the other two.

5. State and prove the principle of the "conservation of areas." Shew that it holds in the case of a body acted on by gravity, moving on a surface of revolution of which the axis is vertical.

6. Shew how to find the altitude of a station above the Earth's surface by means of the barometer.

7. Find the condition among the forces acting on a fluid that equilibrium may be possible. Shew that it is satisfied whenever the forces tend to fixed centres and are functions of the distance.

8. A small pencil is refracted through a prism in a plane perpendicular to its edge; shew that when the deviation is a minimum the pencil at emergence will diverge from a point.

Why is this position of the prism necessary in order to shew the fixed lines of the spectrum?

9. Shew that an eye-piece of two lenses, in a large astronomical telescope, will be achromatic if the interval between the lenses is half the sum of their focal lengths.

Point out the difference in the methods of achromatizing object-glasses and eye-pieces.

10. An orbit similar and equal to the apparent orbit of P round S in motion, may be described round S fixed by the action of the same central force.—(NEWTON. Section XI. Prop. 58.)

What use does Newton make of this proposition?

11. Compare the periodic angular errors in different systems of three bodies.

Does the proportion hold in the case of the Moon and a satellite of Jupiter?

12. Shew how a small error of deviation in a transit instrument may be detected and corrected. Calculate its effect on the time of transit of a given star.

13. How did Bradley separate the effects of nutation and aberration? Shew how to find the coefficient of aberration, without knowing the velocity of light.

14. Find the position of the ecliptic in the heavens at any time with reference to the meridian and horizon of any place.

In what astronomical calculations is this requisite?

THURSDAY, *January* 5.

PROBLEMS.

1. IF $y = e^{\frac{1}{1-\log x}}$, and $z = e^{\frac{1}{1-\log y}}$, prove that $x = e^{\frac{1}{1-\log z}}$.

2. In a quadrant of a circle two semicircles are inscribed, their diameters coinciding with the radii of the quadrant; shew that the area common to the two semicircles is equal to that portion of the area of the quadrant which is outside the semicircles.

3. From a given point outside two given circles, which do not meet, to draw a straight line, such, that the portions of it intercepted by each circle shall be respectively proportional to their radii.

4. If two parallel lines be drawn at an inclination θ to the axis of x through two points whose co-ordinates are $a\ b$ and $a'\ b'$; shew that the distance between these lines is $(b'-b)\cos\theta-(a'-a)\sin\theta$. Hence determine the rectangle whose sides pass through four given points, and whose area is given.

5. A straight line is drawn from a point whose co-ordinates are a b, so as to touch the circle ($x^2+y^2=r^2$), and to cut off a portion (c) from the axis of y; obtain an equation for determining c in terms of a b and r.

Find the locus of the intersection of two tangents to a circle, such, that the rectangle under the portions (measured from the centre) cut off by them from a given diameter is always constant.

6. Shew that $e^{e^{\cdot^{\cdot^{\cdot}}}}$ (to x indices) $= \log^{-x}(e)$.

If $\log^y \{f(x)\} = f(x-y)$, and $f(0)=e$, determine $f(x)$.

7. If k be the last term of an equation of the n^{th} degree, whose roots are in geometric progression, shew that $k^{\frac{1}{n}}$ is one of the roots, n being odd. Shew that, in a similar manner, one root of an equation of odd degree, whose roots are either in arithmetic or harmonic progression, may be found.

8. One end of a beam, whose weight is W, is placed on a smooth horizontal plane; the other end, to which a string is fastened, rests against another smooth plane inclined at an angle (α) to the horizon; the string passing over a pulley at the top of the inclined plane hangs vertically, supporting a weight (P). Shew that the beam will rest in all positions if a certain relation hold between P W and α.

9. A cylinder rests with its base on a smooth inclined plane, a string attached to its highest point, passing over a pulley at the top of the inclined plane, hangs vertically and supports a weight; the portion of the string between the cylinder and the pulley is horizontal: determine completely the conditions of equilibrium.

10. A closed vessel filled with an elastic fluid is let fall; determine what the law of density of the fluid must be, in order that all its particles may be at rest, relatively to each other, during the motion.

11. A heavy uniform rod, suspended at its highest point, is capable of oscillating freely in a vertical plane; each particle of it is attracted by a force emanating from a centre situated at a height (a) directly above the point of suspension, a being also the length of the rod. If μr be the law of the force, shew that the equilibrium is stable or unstable according as μa is less or greater than g.

12. A particle attached to one end of an elastic string moves on a smooth horizontal plane, the other end of the string being fixed to a point in that plane. If the path of the particle be a circle, shew that $r^2 \propto \frac{ra}{r-a}$, a being the natural, r the stretched length of the string, and τ the periodic time.

If the orbit be nearly circular, determine the angle between the apsides.

13. A circle with its plane vertical rolls along a given curve on a horizontal plane: if x_1 y_1 be the co-ordinates of the point of contact of the circle and curve, and x y z the co-ordinates of any fixed point P on the circle, shew that

$$(x-x_1)\,dx+(y-y_1)\,dy+z\,dz=0,$$
$$(x-x_1)^2+(y-y_1)^2+z^2-2az=0,$$

a being the radius of the circle. Hence shew how to find the equation to the curve generated by P, and take for example the common cycloid.

14. Shew that

$$\frac{d^{2n}}{dx^{2n}}\left\{(a^2+x^2)^{n-\frac{1}{2}}\right\}=\left\{1\,.\,3\ldots(2n-1)\right\}^2 a^{2n}(a^2+x^2)^{-n-\frac{1}{2}}.$$

15. If A be the south azimuth of a given vertical plane, c the colatitude, Δ the north polar distance of a star, h its hour angle when it crosses the given vertical plane before noon, 2τ the time (expressed in degrees) between its forenoon and afternoon transit across that plane: shew that

$$\tan(\tau-h)\cos c=\cot A,\quad \cos\tau\tan\Delta=\tan c\cos(\tau-h).$$

16. Hence shew, that the latitude of a place, the position of the meridian, and the polar distance of any star may be found, by means of a clock, and two transit instruments placed in vertical planes at right angles to each other. Must we correct for refraction in this method?

How would you place the transits in vertical planes at right angles to each other?

17. A Screw of Archimedes is capable of turning freely round its axis, which is fixed in a vertical position; a heavy particle is placed at the top of the tube and runs down through it; determine the whole angular velocity communicated to the screw.

18. A thin hemispherical shell, whose radius is a, rests on a smooth horizontal plane; a particle, of the same mass as the shell, is placed at the lowest point of its internal surface, which is smooth: shew that, if the shell be projected with a horizontal velocity $2\sqrt{ga}$, the particle will ascend just as high as the centre of the shell, and then descend.

19. If the velocity of projection be greater than $2\sqrt{ga}$, determine completely the nature of the motion so far as velocities are concerned.

20. A and C are the moments of inertia of the terrestrial spheriod about its equatorial and polar diameters respectively: shew that

$$\frac{C-A}{A} = \frac{\int_0^{a_1} \rho\, d(a^5\epsilon)}{\int_0^{a_1} \rho\, d(a^5)}.$$

21. If $\dfrac{1}{\sqrt{1-2\mu h+h^2}} = P_0 + P_1 h + P_2 h^2 + \ldots P_n h^n + \ldots,$

shew that $\dfrac{d}{d\mu}\left\{(1-\mu^2)\dfrac{dP_n}{d\mu}\right\} + n(n+1)P_n = 0,$

$$\frac{d^n(P_n)}{d\mu^n} = 1 \cdot 3 \cdot 5 \ldots (2n-1),$$

and if $P_n = C(\mu^n + A_1\mu^{n-1} + A_2\mu^{n-2}\ldots)$ determine C, A_1, $A_2\ldots$

22. If $\alpha\,\beta\,\gamma$ be the displacements, parallel to the axes of $x\ y\ z$, of any particle of the luminiferous ether from its equilibrium position $(x\ y\ z)$, and if

$$\alpha = a\cos\phi + (qc - sb)\sin\phi,\quad \beta = b\cos\phi + (sa - pc)\sin\phi,$$
$$\gamma = c\cos\phi + (pb - qa)\sin\phi,\ pa + qb + sc = 0,\ p^2 + q^2 + s^2 = 1,$$
$$\text{and } \phi = k(vt - px - qy - sz),$$

$a\ b\ c\ p\ q\ s\ k$ and v being constants: shew what each of these constants represents, and prove that the light is circularly polarized.

23. Conceive a system of surfaces of such a nature, that a normal at any point of one of them is a normal to all the rest; let $u = 0$ be the equation to one of these surfaces, let the normal at any point P be the axis of z, and let the distance of the origin from the point P be a harmonic mean between the greatest and least radii of curvature at that point. Then shew that at the point P

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = \frac{1}{z}\frac{d^2(uz)}{dz^2}.$$

24. One of the equations, which determine the vibratory motion of the particles or air, may be put in the form

$$\frac{d^2\alpha}{dt^2} = A\left(\frac{d^2\alpha}{dx^2} + \frac{d^2\alpha}{dy^2} + \frac{d^2\alpha}{dz^2}\right).$$

$x\ y\ z$ being the co-ordinates (referred to any rectangular axes) of a particle in its equlibrium position, and α the displacement of that particle parallel to the axis of x at any time t during the motion: shew that, if the wave-surfaces be of the nature of the surfaces in (23), the value of α is

$$\alpha = \frac{1}{n}\left\{f(\sqrt{A}\,.\,t - n) + \phi(\sqrt{A}\,.\,t + n)\right\},$$

n being that portion of the normal at any point which is a harmonic mean between the greatest and least radii of curvature at that point. Interpret this result.

FRIDAY, *January* 6.

QUESTIONS IN PURE MATHEMATICS.

1. TRIANGLES and parallelograms of the same altitude are to one another as their bases.

What limitation is there to the alternation of a *geometrical* proportion?

2. If a solid angle be contained by three plane angles, the sum of any two of them is greater than the third.

3. If an integer be prime to two other integers, it is prime to their product.

Shew that an integer cannot be resolved into two different sets of prime factors.

4. Defining a^m to be an operation performed on a, which satisfies the condition

$$a^m \cdot a^n = a^{m+n}, \quad a^1 = a,$$

determine its meaning when m is a positive integer, a negative integer, or a fraction.

On the same principle determine a meaning of $(-)^{\frac{1}{n}}$.

5. The sum of the m^{th} powers of the roots $x^n - 1 = 0$ is equal to zero, unless m be a multiple of n, in which case the sum is equal to n.

6. Define an ellipse, and from your definition deduce the polar equation referred to the focus as pole.

Shew that, when the curve is referred to rectangular co-ordinates, the focus being the origin, the radius vector is a rational and integral function of the co-ordinates of its extremity.

7. Determine the equations to the asymptotes of the curve

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0,$$

shewing what is the necessary condition that it may have asymptotes.

8. In spherical triangles prove the formula

$$\cot A \,.\, \sin C = \cot a \,.\, \sin b - \cos C \,.\, \cos b,$$

adapt this to logarithmic computation, and shew that the introduction of a subsidiary angle corresponds to dividing the triangle into two right-angled triangles.

9. Find the equations to a straight line passing through a given point and perpendicular to a given plane. Determine also the length of the perpendicular intercepted between the point and the plane.

10. Having given $u = 0$ a relation between y and x, shew how to find the differential coefficient of y with respect to x.

Find $\frac{dy}{dx}$ from $y\epsilon^{ny} = ax^m$, and from $y = \cos^{-1}\sqrt{\frac{a^2 - x^2}{b^2 - x^2}}$;

and if $y = \cos mx$, find $\frac{d^n y}{dx^n}$.

11. Apply the method of limits to find the equation to a straight line which touches a curve at a given point.

If x_0, y_0 be the portions of the axes cut off by a tangent at an apse, r the distance of the point of contact from the origin, shew that

$$\frac{1}{{x_0}^2} + \frac{1}{{y_0}^2} + \frac{1}{r^2}.$$

12. Shew how the maximum and minimum values of a function of one variable may be determined.

If this method be applied to find those conjugate diameters in an ellipse of which the sum is a maximum or a minimum, it appears to fail for the latter case:—explain the cause of this.

13. Integrate $\frac{dx}{(a^2 + x^2)^{\frac{3}{2}}}$, $\frac{dx}{(x^2 + a^2)\sqrt{x^2 + 5a^2}}$, $\frac{dx}{5 + 3\cos x}$.

Find the value of $\int_0^{\frac{\pi}{4}} dx\,(\tan x)^n$.

14. What is meant by integration between limits? When the function to be integrated changes its sign between the limits, how is the true value of the definite integral to be found?

Trace and find the whole area of the curve

$$r = a(2\cos\theta + 1).$$

FRIDAY, *January* 6.

PROBLEMS.

1. IF any number (n) can be resolved into the sum of (q) squares, $2(q-1)n$ can be resolved into the sum of $q(q-1)$ squares.

2. If r_1, r_2, r_3 be the radii of three small circles of a sphere (radius r) which touch one another in P, Q, R, and A, B, C be the angles of the spherical triangle formed by joining their centres.

$$\text{area } (PQR) = (A\cos r_1 + B\cos r_2 + C\cos r_3)\,\pi r^2.$$

3. $\tan\theta + \tan\left(\frac{2\pi}{m} + \theta\right) + \ldots + \tan\left(2\frac{m-1}{m}\pi + \theta\right) = \frac{m\sin m\theta}{\cos m\theta - S_m(-1)^{\frac{m}{2}}}$, where $S_m = \frac{1}{2}$. the sum of the m^{th} powers of the roots of $z^2 - 1 = 0$.

4. The stereographic projection of a cube on a plane perpendicular to its diagonal (the pole being in the diagonal produced) is an equilateral hexagon, whose angles are alternately greater and less. If the pole of projection be at a distance from the cube, equal to its diagonal, the sines of two adjacent angles of the hexagon are as 8 : 5.

5. Two tangents (a, b) to a parabola intersect in P at an angle (ω), and a circle is inscribed between the tangents and the curve, shew that the distance of its centre from P is

$$= \frac{ab}{(a+b)\sec\frac{\omega}{2} + \sqrt{ab}\tan\frac{\omega}{2}}.$$

6. Two heavy triangular lamniæ have a common horizontal base, about which they are capable of revolving freely; and their vertices are joined by an elastic string. Find the position of unstable equilibrium in which they will rest; and the increase of length of the string.

7. The circular base of a cone remains constant, while the vertex moves in a segment of a circle described on a diameter of the base, having its plane at right angles to the base; find the surface on which lie all the sub-contrary sections of the cone.

8. A small pencil of rays enters a reflecting sphere, and after n internal reflexions emerges through the same aperture; find the angle of incidence, and the position of the primary focal line at emergence.

9. A prolate spheroid rests symmetrically on two rods equally inclined to the horizon, their point of concourse being lowest; find the position of equilibrium, and the time of a small oscillation.

10. From every point of the surface (1) $4z(lx^2 + l'y^2 + ll'(x^2+y^2) = 0$ as vertex, enveloping cones are drawn to the paraboloid (2) $ll'z - (lx^2 + l'y^2) = 0$; shew that the planes of contact all touch the surface $4z(lx^2 + l'y^2) - (l^2x^2 + l'^2y^2) = 0$: also shew that the surface (1) contains the directrices of all the sections of (2) made by planes through its axis.

11. A solid generated by the revolution of the curve $y \propto x^{\frac{n-1}{2}}$ round the axis of x, floats in a fluid with a portion (h) of the axis immersed. If the solid be depressed through

$(n^{\frac{1}{n-1}} - 1)h$, it will on its return just emerge from the fluid, (neglecting the motion of the fluid.)

12. In a curve described by a body in one plane by the action of a central force, the angle between the radius vector and the tangent varies as the time. Find the curve and law of force.

13. The Sun's light is reflected by a mirror to a given point Q. If PN the North Polar Distance of the normal to the mirror be produced to R so that $PR = 2PN$, R traces out a circle round Q.

14. Shew that $\frac{\theta}{\theta'} = 2 \cdot \left(\frac{\sin\theta}{\sin\theta'} - \frac{\sin 2\theta}{\sin 2\theta'} + \frac{\sin 3\theta}{\sin 3\theta'} - \ldots\right)$.

15. If a curve of the fourth order have three double points, the six tangents at those points all touch a conic section.

16. If (ρ) be the radius vector (p) the perpendicular from the origin on the tangent, (s) the arc of a curve whether plane or of double curvature, $p\frac{dp}{d\rho} = \frac{p^2}{\rho}\rho^2 \cdot \frac{d^2\rho}{ds^2}$ and in a curve of double curvature, if (R) is the radius of absolute curvature, (ρ') the distance of the centre of absolute curvature from the origin, then

$$\rho^2\left(\frac{d\rho}{dp}\right)^2 = \frac{2R^2p^2}{R^2 + p^2 - \rho'^2}.$$

17. The sum of the (m) least roots of the equation $x^m - 1 = \epsilon^x$ is a series of which the $(r)^{\text{th}}$ term is $\frac{1}{r}\frac{\Delta^r 0^{rm-1}}{1.2\ldots(rm-1)}$.

18. An ellipsoid rests with its least axis (c) vertical on a rough horizontal plane: when slightly disturbed in any manner, find the path of the centre of gravity. Shew that the coexistent oscillations in principal planes cannot be synchronous, but that the principal planes will attain maximum amplitudes together when

$$\frac{\text{number of oscillations in plane } (a, c)}{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b, c)} = \left\{\frac{a^2 - c^2}{b^2 - c^2} \cdot \frac{b^2 + 6c^2}{a^2 + 6c^2}\right\}.$$

19. If $G(u_x)$ be the generating function of u_x,

$$G(\log_e x) = \frac{\log_e(1-t)}{\log_e(t)}, \quad G(\cos x\phi) = \frac{t\cos\phi - t^2}{1-2t\cos\phi+t^2},$$

$$G\tan^{-1}x =$$

$$\frac{1}{2\log_e\left(\frac{1}{t}\right)}\int\left(\frac{t\sqrt{-1}}{t}\int\left(\frac{t-\sqrt{-1}}{1-t}dt + \frac{t-\sqrt{-1}}{t}\int\frac{t\sqrt{-1}}{1-t}dt\right)\right)dt.$$

20. SP, SP' are two radii vectores of a curve, SY, SY' are corresponding perpendiculars from the origin on the tangents at P, P'. Find the curve when if $SP = SY'$ then shall also $SP' = SY$.

21. A sphere described from the origin with radius $\frac{abc}{ac+ab+bc}$ will touch the surface $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1$ in points which are *umbilici*.

22. A ray of light polarized in the plane of incidence enters a crystal cut perpendicularly to its axis, and is separated into two elliptically polarized rays of opposite but equal ellipticities, find the intensity after reflexion from an analyzing plate.

23. The attraction of an ellipsoidal shell on an external particle is in the direction of the axis of the enveloping cone, whose vertex is the given particle.

24. If the Earth had no motion of rotation about its axis, but yet preserved the form of an oblate spheroid, the action of the Sun would cause a uniform progression of the mean equinox, and the true equinox would make small oscillations about the mean, whose period would be three months nearly.

SATURDAY, *January* 7.

PROBLEMS.

1. A CERTAIN fraction becomes 1, when 3 is added to its numerator; and becomes $\frac{1}{2}$ when 2 is added to its denominator: what fraction is it?

2. Bisect a right-angled triangle by a line perpendicular to its hypothenuse.

3. A, B, C, A', B', C' are points on the circumference of a circle; if the lines AB, AC be respectively parallel to $A'B'$, $A'C'$, shew that BC' is parallel to $B'C$.

4. If two circles, whose radii are a and b, touch each other externally, and if θ be the angle contained by the two common tangents to these circles; shew that

$$\sin\theta = \frac{4(a-b)\sqrt{ab}}{(a+b)^2}.$$

5. One extremity of a straight rod moves in a circle, and the other in a straight line passing through its centre; determine the equation to the path of any point of the rod, and discuss it. Determine the same when the rod does not pass through the centre.

6. Shew that the equation to the locus of the middle points of all chords of the same length ($2c$) in an ellipse is

$$c^2 \cdot \frac{a^2y^2 + b^2x^2}{a^4y^2 + b^4x^2} + \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0.$$

7. AP, BQ are two arms moving round the fixed centres A and B, and the points P and Q are connected by a link (i. e. a rod) PQ; shew that the angular velocities of the arms AP, BQ are inversely proportional to the segments into which the link divides the line of centres (i. e. AB).

8. Ellipsoids defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, whose axes satisfy the condition $\frac{k^2}{a^2} + \frac{l^2}{b^2} = 1$, all touch a certain straight line in the same point; and if the straight line meet the plane of xy in the point (a', b'), the co-ordinates of the projection of the point of contact on that plane are $\left(\frac{k^2}{a'}, \frac{l^2}{b'}\right)$.

9. A heavy elastic string is laid upon a smooth double inclined plane in such a manner as to remain at rest; find how much the string is stretched.

10. Two wheels revolving uniformly in the same plane are suddenly brought in contact, and their axes are kept fixed; the friction being sufficient to prevent all sliding, determine what alteration is made in the angular velocities.

11. The time of transit of the Moon's diameter across any meridian circle of an equatoreal is nearly unaffected by parallax.

12. Two gases whose volumes, temperatures and pressures are known, are mixed under the standard temperature and pressure: the mixture is observed to contract and cool, the pressure remaining the same; the reduced temperature being known, the whole mass is raised to the standard temperature and its volume is noted. Find the contraction of the mixture at first.

13. In a vessel full of fluid, revolving uniformly about a vertical axis, a very small sphere, lighter than the fluid, is attached by a string of given length to a given point in the base, being wholly immersed. Determine the position of the sphere and the tension of the string, when the sphere is at rest relatively to the fluid.

14. An ellipsoid revolves about a line through its centre in

the plane of (ab) inclined at an angle θ to (a): if $e^2 = 1 - \frac{c^2}{a^2}$, $e'^2 = 1 - \frac{c^2}{b^2}$ the spheroid which envelopes it in every position is

$$e^2 e'^2 (x + y \tan\theta)^2 = (x^2 + y^2 + z^2 - c^2)(e^2 + e'^2 \tan^2\theta).$$

15. Having given the radii of the outer and inner shaft of a winding staircase, and its constant inclination; find the length of the longest beam that can be carried up it.

16. An elliptic board has the extremity of its minor axis fixed; it is struck in a direction perpendicular to its plane through one focus. If the eccentricity $= \frac{1}{\sqrt{5}}$, the axis of initial rotation will pass through the other focus.

17. Suppose that the Moon moves in the plane of the ecliptic, that the Sun's apparent orbit is a circle, and that

$$u = a\{1 + m^2 \cos 2(\theta - \theta')\},$$
$$\theta = pt + \frac{11}{8} m^2 \sin 2(\theta - \theta'):$$

then shew that the Moon, in addition to the Earth's attraction, is acted on by a force which always passes through the Sun, neglecting terms multiplied by powers of m above the square.

18. The Moon's libration in longitude (u) is given by the equation

$$\frac{d^2u}{dt^2} + 3m^2 \frac{B-A}{C} u = 3m^2 \frac{B-A}{C} \Sigma H \sin(ht + h').$$

Integrate this equation, and examine the effect of the several terms. Shew that observations of (u) might afford a means of determining $\frac{B-A}{C}$.

SATURDAY, *January* 7.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

1. IN the conversion of a vulgar fraction into a decimal, find the form of the fraction in order that the division may terminate. When the division does not terminate find the greatest number of figures which there can be in the period.

2. The successive convergents of a continued fraction are alternately greater and less than the true value, each being nearer to the truth than the one preceding it.

3. Expand $\cos n\theta$ in terms of the powers of $\cos\theta$, and discuss the form of the result according to the nature of n.

4. Find the equation to the line joining the points of contact of two tangents drawn to an ellipse from any point. If the point move along a straight line, shew that the chord of contact will always pass through a fixed point.

5. Shew how to determine the existence of points in which different branches of a curve meet. How do you distinguish points in which branches cut from those in which they touch each other?

6. Shew that the hyperboloid of one sheet may be generated by the motion of a straight line, no three positions of which are parallel to the same plane.

7. When any number of forces act on a rigid body, find the condition that they may have a single resultant.

Ex. A cube acted on by four forces, one in a diagonal, the others in edges, no two of which are in the same plane, and which do not meet the diagonal.

8. If a body be moving in a plane curve, prove that the resolved effective forces in the tangent and normal are $\frac{d^2s}{dt^2}$ and $\frac{v^2}{\rho}$

respectively; s being the arc described, v the velocity, ρ the radius of curvature.

9. Shew that, when a body floating in a fluid slightly disturbed, neither vertical nor angular oscillations can exist separately, unless the line joining the centres of gravity of the body and fluid displaced in equilibrium passes through the centre of gravity of the plane of floatation.

10. Trace the relative positions of the foci of incident and refracted pencils in a convex lens, and find when their distance is a minimum.

What practical application is made of the result obtained?

11. Integrate the equations

$$xy\frac{dy}{dx} + x^2 = y^2, \qquad y\left(\frac{dy}{dx}\right)^2 + (2x-a)\frac{dy}{dx} - y = 0.$$

Find the value of $\int_0^\infty dx\, \epsilon^{-x^2}$.

12. Having given one of the principal axes through any point of a body, find the other two.

Find the principal axes passing through the right angle of a right-angled triangle, and the moments of inertia round them.

13. An uniform beam is supported on two props at its extremities: if one of the props be suddenly removed, find the instantaneous pressure on the other.

14. Find the inclination of a planet's orbit from observations made on the planet when the Earth is in the line of nodes.

An element of the orbit is required in this proposition: state what observations are required to determine it.

15. A series of waves diverging from a point falls on a prism, of which the angle is very obtuse: assuming the intensity of light at a point M of the screen to be given by the formula

$$\frac{4c^2}{(a+b)^2}\cos^2\left(\frac{2\pi}{\lambda}\frac{(\mu-1)\,a\sin\alpha}{a+b}OM\right),$$

interpret its meaning, and explain why the number of fringes is much smaller for heterogeneous than for homogeneous light.

MONDAY, *January* 9.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

1. GIVE an independent definition of the tangent of an angle; and trace its sign and magnitude as the angle increases from 0^0 to 360^0.

What is the value of tan 195^0?

2. A given force supports a given weight on a rough inclined plane. Find the inclinations of the force to the plane between which equilibrium may subsist.

Mention some instances (1) where friction is necessary to the efficient action of a force, (2) where it adds to the resistance to be overcome.

3. A weight hanging freely draws another up an inclined plane, by means of a cord passing over a pulley and parallel to the plane. Find the time of describing a given space from rest.

4. If an equation contain equal roots, shew how they may be found.

Decompose $\sin x - 1 = 0$ into its factors.

5. Find the lens of minimum aberration for parallel rays.*

Why may we not find the ratio of the radii of the lens by assuming the aberration to vanish instead of being a minimum?

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* Assuming that for parallel rays,

$$\frac{1}{v} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) + \frac{1}{\{\mu(\mu-1)\}^2}$$

$$\{\mu.(\mu+2)x^2 - 4\mu.(\mu^2-1)x + 4\mu^2(\mu-1) - \mu + 2\}\frac{y^2}{8f^3},$$

$$\text{where } \frac{1}{r} = \frac{x+1}{2(\mu-1)f}.$$

6. Find the co-ordinates of the centre of pressure of a plane surface immersed in a fluid.

Ex. A parabola has its vertex in the surface of a fluid and its axis vertical; find the centre of pressure of a segment cut off by a line inclined at 45^0 to the axis, and passing through the vertex.

7. Find the equation to the osculating plane of a curve of double curvature; and shew that it is perpendicular to the line of intersection of consecutive normal planes.

8. If u, v be functions of x, prove the theorem

$$\frac{d^n(uv)}{dx^n} = v\frac{d^nu}{dx^n} + n\cdot\frac{dv}{dx}\cdot\frac{d^{n-1}u}{dx^{n-1}} + \frac{n\cdot(n-1)}{1\cdot 2}\frac{d^2v}{dx^2}\cdot\frac{d^{n-2}u}{dx^{n-2}} + \ldots$$

Ex. Shew that $\left(\frac{d}{dx} - a\right)^n f(x) = \epsilon^{ax}\left(\frac{d}{dx}\right)^n \epsilon^{-ax} f(x)$.

9. Shew how the latitude may be found by observations on the pole star out of the meridian.

10. Why does the rotation of a hoop keep it from falling?

11. Prove the formulæ:

$$(1)\quad u_{x+n} = \epsilon^{n\frac{d}{dx}} u_x, \qquad (2)\quad \Delta^n u_x = (\epsilon^{\frac{d}{dx}} - 1)^n u_x.$$

Shew that the coefficient of $\frac{d^{n+r}u_x}{dx^{n+r}}$ in the second side of (2) is

$$\frac{\Delta^n 0^{n+r}}{1.2\ldots(n+r)}.$$

12. Having given the partial differential coefficients of a function with respect to any variables, shew how to change them to any other set of variables connected with the first by given conditions.

If (x, y, z), (x', y', z') be two sets of co-originate rectangular co-ordinates, (u) any function of them: shew that

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} + \frac{d^2u}{dx'^2} + \frac{d^2u}{dy'^2} + \frac{d^2u}{dz'^2}.$$

MONDAY, *January* 9.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

1. FIND the present value of an annuity to commence a year hence and to continue for (n) years, allowing compound interest.

What is the income corresponding to an income tax of £13. 2*s*. 6*d*., at the rate of 7 pence in the pound?

2. Prove that $\frac{\sin\theta}{\theta}$, $\frac{\tan\theta}{\theta}$, $(\cos\theta)^{\frac{a}{\theta}}$ have unity for their limit when θ is indefinitely diminished.

If (n) be the number of seconds in θ, shew that $n = \frac{\theta}{\sin 1''}$.

3. Determine the radius of curvature at any point of a curve. Shew that in the curve, $y = \frac{c}{2}(\epsilon^{\frac{x}{c}} + \epsilon^{-\frac{x}{c}})$, the ordinate at any point is a mean proportional between the radius of curvature there, and that at the point $(x = 0.)$

4. Find the time of a small oscillation of a rigid body round a horizontal axis. Shew that the centres of suspension and oscillation are convertible.

5. Given the displacements of the true place of the pole measured from its mean place along the solstitial colure and perpendicular to it, find the effect on the right ascension and north polar distance of a given star.

6. Prove that

$$\Sigma(u_x v_x) = u_x \Sigma v_x - \Sigma(\Delta u_x \Sigma v_{x+1}),$$ and find $\Sigma(x^3 a^x)$.

Prove that $u_x = u_0 + x \cdot \Delta u_0 + \frac{x(x-1)}{1 \cdot 2}\Delta^2 u_0 + \ldots\ldots$, and

shew that the sum of the series $\frac{1}{\sin\theta} + \frac{1}{\sin 2\theta} + \frac{1}{\sin 4\theta} + \ldots\ldots$ to x terms is equal to $\cot\frac{\theta}{2} - \cot 2^{x-1}\theta$.

7. Calculate the approximate values of the central and tangential disturbing forces of the Sun on the Moon. (NEWTON, Book III. Prop. 25.)

8. Find an expression for the height of the tide at a given place, when the Sun and Moon are in the equator; and shew how the Moon's mass may be derived from observations of the heights of spring and neap tides.

9. Assuming that

$$\frac{dx}{dt} = \omega_2 z - \omega_3 y, \quad \frac{dy}{dt} = \omega_3 x - \omega_1 z, \quad \frac{dz}{dt} = \omega_1 y - \omega_2 x;$$

find the three differential equations which determine $\omega_1, \omega_2, \omega_3$.

If the body be acted on by no forces, shew that

$$A\omega_1^2 + B\omega_2^2 + C\omega_3^2 = \text{constant}.$$

10. In forming the differential equations for the Moon's motion to the second approximation, terms of the third order of small quantities are preserved; explain the necessity of this, and state the arguments of the terms thus introduced into the Moon's longitude.

11. If $V = f\left(x, y, z, \frac{dy}{dx}, \frac{dz}{dx}\right)$, find the condition that $\int V dx$ may be a maximum or minimum.

Between two fixed points in space draw a curve of given length, such that its projection on a given plane may subtend at a given point in the plane a sector of maximum area.

TUESDAY, *January* 10.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

1. SHEW that there cannot be more than five regular solids.

2. Shew how the perturbation of the motion of a pendulum may be determined by the method of the variation of parameters.

Ex. A pendulum is acted on by a small constant friction at the point of support: shew that it does not affect the time, but diminishes the arc of vibration.

3. Integrate $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = x^2$, and $u_{x+1} = a(u_x)^n$.

If $z = \phi\left(\frac{y}{x}\right)$, and $\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} = 0$, find the form of ϕ.

Shew that $\int_0^1 dx\ x^{m-1}(1-x)^{n-1} = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.

4. Out of a set of cards, numbered from 1 to 10, a card is drawn and replaced; after six such drawings find the probability that the sum of the numbers drawn is 24.

5. Having given

$$0 = \frac{d^2s}{d\theta^2} + s + \tfrac{3}{2}m^2k\sin(g\theta - \gamma) - \tfrac{3}{2}m^2k\sin\{(2-2m-g)\theta - 2\beta + \gamma\};$$

integrate this equation, and explain the effect of the several terms.

If $\delta\gamma$ and δk be the variations of γ and k introduced by the last term, shew that

$$(\delta\gamma)^2 + \left(\frac{\delta k}{k}\right)^2 = \text{const.}$$

6. Shew that the radius of curvature of any section of a surface is equal to the projection, on the cutting plane, of the radius of curvature of the normal section which passes through the same tangent-line.

7. If V be the sum of the masses of any number of attracting particles, each divided by its distance from a point (a, b, c), shew that

$$\frac{d^2V}{da^2} + \frac{d^2V}{db^2} + \frac{d^2V}{dc^2} = -4\pi\rho, \text{ or } = 0,$$

according as (a, b, c) is or is not a part of the attracting mass, ρ being the density at that point.

8. Deduce the common laws of reflection and refraction from the undulatory theory of light, shewing that when the angle of incidence exceeds the critical angle there is no refracted ray. What difficulty attends the explanation of dispersion on the undulatory hypothesis?

9. Find the disturbing forces of one planet upon another, and prove that

$$\left(\frac{dr_1}{dt}\right)^2 + r_1^2\left(\frac{d\theta_1}{dt}\right)^2 - \frac{2\mu}{r_1} + 2\int\frac{d(R)}{dt}\,dt = C.$$

What is the object of substituting $n\frac{dR}{d\epsilon}$ for $\frac{d(R)}{dt}$?

TUESDAY, *January* 10.

QUESTIONS IN PURE MATHEMATICS AND NATURAL PHILOSOPHY.

1. DISTINGUISH between convergent and divergent infinite series. The series $u_0 + u_1 + \ldots u_n + \ldots$ is convergent or divergent according as $\frac{u_{n+1}}{u_n}$ tends to a limit less or greater than unity, when (n) is indefinitely increased.

Shew that the expansion of ϵ^x is convergent for all values of x.

2. Explain the nature of developable surfaces, and find their differential equation. Shew that the osculating plane of a curve of double curvature will in its consecutive positions envelop a developable surface, of which the generating lines are the tangents of the curve.

3. Investigate generally the motion of a vibrating string.

Find the ratio of the numbers of longitudinal and transverse vibrations which constitute the fundamental notes produced by each kind of vibration.

4. Find the effects of the Moon's attraction on the obliquity of the ecliptic during a long period, assuming that the motion of the pole in a sidereal revolution of the Moon is

$$\frac{6\pi^2 . B . \sin I' \cos I'}{T'^2 \omega (n+1)} .$$

5. In Kater's pendulum calculate the effect of shifting the small weight. Explain the principle of Kater's method, and point out the use of the small weight.

6. Shew that in fluid motion the particles move in normals to the surface represented by the equation

$$udx + vdy + wdz = 0.$$

7. State and prove Ivory's theorem for making the attraction of an ellipsoid on an external particle depend on that of another ellipsoid on an internal particle.

8. Shew that in the developement of the disturbing function for two planets, the principal term with the argument $(pn - qn')$ $t + Q$ is of the order $(p - q)$.

9. In a doubly-refracting crystal, there are two directions of displacement of a particle in a given plane front of a wave,

for which the force acting upon it, perpendicularly to the direction of the displacement, is also perpendicular to the same plane front. Prove this, and shew that these directions are at right angles to each other.

Are the positions of the optic axes affected by the temperature of the crystal?

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